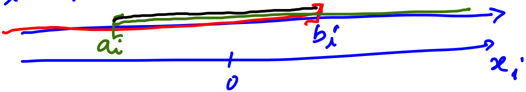
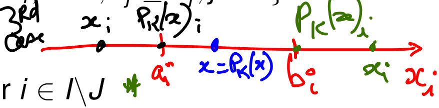
Projection on an intersection of half spaces

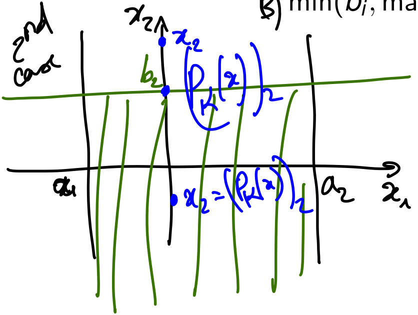
in direction i K looks like

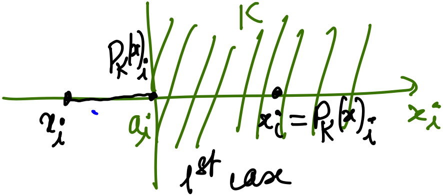


Supposons que $K = \{x \in \mathbb{R}^n, x_i \geq a_i, i \in I, x_j \leq b_j, j \in J\}$ avec $I, J \subset \{1, \ldots, n\}$. On a alors



$$P_K(x)_i = \begin{cases} 1)\ \max(a_i, x_i), \text{ pour } i \in I \backslash J \quad * \\ 2)\ \min(b_i, x_i), \text{ pour } i \in J \backslash I \\ 3)\ \min(b_i, \max(a_i, x_i)), \text{ pour } i \in I \cap J \quad \leftarrow \text{3rd case} \end{cases}$$





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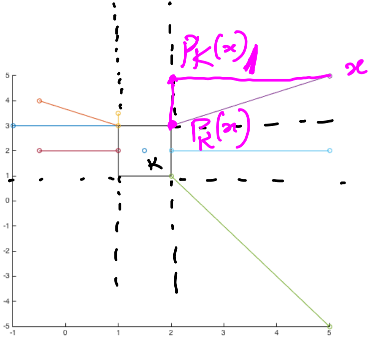
$$P_K(x)_i = \begin{cases} \max(a_i, x_i), \text{ pour } i \in I \backslash J \\ \min(b_i, x_i), \text{ pour } i \in J \backslash I \\ \min(b_i, \max(a_i, x_i)), \text{ pour } i \in I \cap J \end{cases}$$



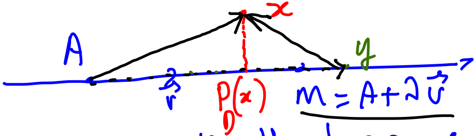
Projection on a line $(A, \vec{v})$

$D = \{ M,\ \overrightarrow{AM} = \lambda \vec{v},\ \lambda \in \mathbb{R} \}$

$A \in \mathbb{R}^n,\ \vec{v} \in \mathbb{R}^n$

in $\mathbb{R}^2$: $ax_1 + bx_2 = c$

in $\mathbb{R}^3$: $\begin{cases} a_1x_1 + b_1x_2 + c_1x_3 = d_1 \\ a_2x_1 + b_2x_2 + c_2x_3 = d_2 \end{cases}$



$M = A + \lambda \vec{v}$

1) write the minimization Pb to find $P_D(x)$

2) Solve with the Lagrange multiplier theorem

1) $f_x(y) = \|x - y\|^2 = \|\overrightarrow{yx}\|^2$ such that $f_x(P_D(x)) = \min_{y \in D} f_x(y)$

$y \in D \iff y = A + \lambda \vec{v}$: $f_x(y) = \|x - A - \lambda \vec{v}\|^2 = \|\overrightarrow{Ax} - \lambda \vec{v}\|^2$

$= g(\lambda)$

$\overrightarrow{Ay} = \lambda \vec{v} = \overrightarrow{Ax} + \overrightarrow{xy}$

$\min_{\lambda \in \mathbb{R}} g(\lambda) =$

$\overrightarrow{AM} = (M) - (A) = (A) + \lambda \vec{v} - (A) = \lambda \vec{v}$

point M = point $A + \lambda \vec{v}$

Projection on a line $(A, \vec{v})$

$$g(\lambda) = \|\overrightarrow{Ax} - \lambda\vec{v}\|^2 = \|\lambda\vec{v} - \overrightarrow{Ax}\|^2$$

$$g'(\lambda) = 2\langle \vec{v}, \lambda\vec{v} - \overrightarrow{Ax} \rangle$$

$$g'(\lambda) = 0$$

$$\langle \vec{v}, \lambda\vec{v} \rangle = \langle \vec{v}, \overrightarrow{Ax} \rangle$$

$$\lambda^* = \frac{\langle \vec{v}, \overrightarrow{Ax} \rangle}{\|\vec{v}\|^2}$$

$$P_K(x) = A + \lambda^*\vec{v} = A + \frac{\langle \vec{v}, \overrightarrow{Ax} \rangle}{\|v\|^2}\vec{v}$$

$$\overrightarrow{AP_K(x)} = \frac{\langle \vec{v}, \overrightarrow{Ax} \rangle}{\|v\|^2}\vec{v}$$