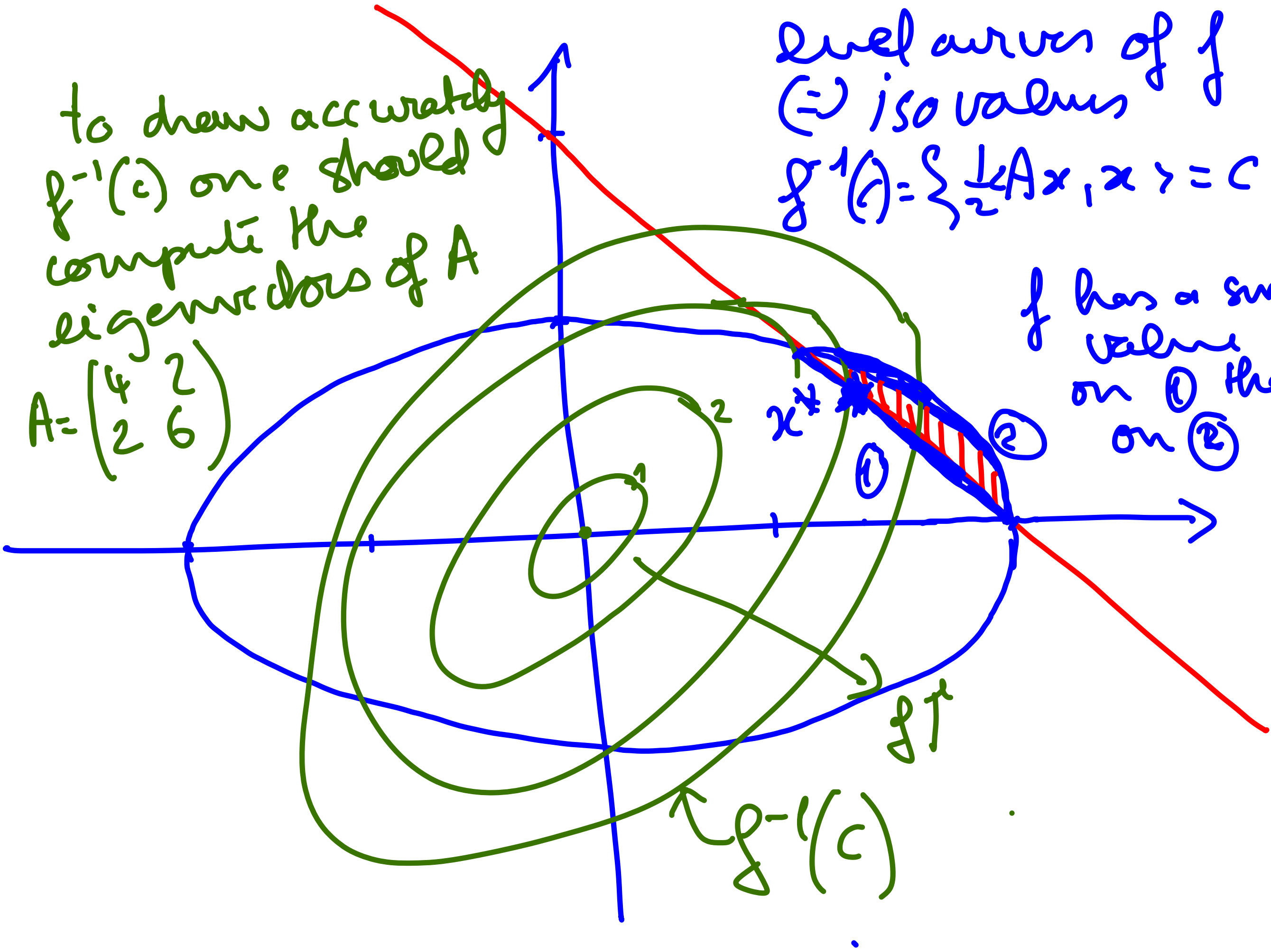
to draw accurately $f^{-1}(c)$ one should compute the eigenvectors of A

$$A = \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix}$$

level curves of f $\iff$ isovalues

$$f^{-1}(c) = \left\{ \tfrac{1}{2}\langle Ax, x\rangle = c \right\}$$

f has a smaller value on ① than on ②



$$f(x) = 2x_1^2 + 3x_2^2 + 2x_1 x_2 \qquad \inf f(x)$$
$$c(x) \leq 0$$
$$c_1(x) = x_1^2 + 4x_2^2 - 1$$
$$c_2(x) = 1 - x_1 - x_2$$

Lagrangian $\quad \ell(x, z) = f(x) + \langle z, c(x) \rangle$

$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \quad \ell(x, z) = f(x) + z_1(x_1^2 + 4x_2^2 - 1) + z_2(1 - x_1 - x_2)$

qualification $\quad JC(x) = \begin{pmatrix} 2x_1 & 8x_2 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} \nabla c_1^T \\ \nabla c_2^T \end{pmatrix}$

if both constraints are active at x^* $\quad Rk\, JC(x^*) = 2$

$2x_1 = -8x_2 \qquad x_1 = -4x_2 \qquad 0 = 1 + 4x_2 - x_2 \qquad x_2 = -\frac{1}{3}$

absurd $\Rightarrow Rk = 2$

if c_1 active only $\quad Rk\ 1$ because $(0,0) \neq x^*$

if c_2 active only $\quad Rk\ 1$ because $(1,1) \neq (0,0)$

1) $$\nabla_x \ell(x,z) = \begin{pmatrix} 4x_1 + 2x_2 \\ 2x_1 + 6x_2 \end{pmatrix} + 2z_1 \begin{pmatrix} x_1 \\ 4x_2 \end{pmatrix} + z_2 \begin{pmatrix} -1 \\ -1 \end{pmatrix} = 0$$

2) $$x_1^2 + 4x_2^2 - 1 \leq 0$$
$$1 - x_1 - x_2 \leq 0$$

3) $$z_1 \geq 0 \qquad z_2 \geq 0$$

4) $$\left.\begin{array}{l} z_1 (x_1^2 + 4x_2^2 - 1) = 0 \\ z_2 (1 - x_1 - x_2) = 0 \end{array}\right)$$

	$z_1 = 0$	$x_1^2 + 4x_2^2 = 1$
z_2		
$x_1 + x_2 = 1$	$\cancel{\text{good case for min}}$	

$$\left(\begin{array}{l} 4x_1 + 2x_2 - z_2 = 0 \\ 2x_1 + 6x_2 - z_2 = 0 \\ 1 - x_1 - x_2 = 0 \end{array}\right.$$

$$2x_1 = 4x_2 \qquad x_1 = 2x_2$$

$$x_2 = \frac{1}{3} \qquad x_1 = \frac{2}{3}$$

$f(x) \in P^2(\mathbb{R})$

$f(x) = ax^2 + bx + c$ $\qquad f''(x) = 2a$

$f(x) = f(0) + f'(0)x + o(|x|)$ TF order 1

$= c + bx + \underbrace{o(|x|)}_{ax^2}$

$f(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + 0$ because $f^{(n)}(x) = 0$ if $n \geq 3$

$= c + bx + \frac{1}{2} 2a\, x^2$

$\Delta = b^2 - 4ac$ $\quad \left(\Delta' = \left(\frac{b}{2}\right)^2 - ac \qquad x = \frac{-b/2 \pm \sqrt{\Delta'}}{a} \right)$

$f(x) = 2x_1^2 + 3x_2^2 + 2x_1 x_2$

$f(\vec{0}) = 0$

$\nabla f(x) = \begin{pmatrix} 4x_1 + 2x_2 \\ 6x_2 + 2x_1 \end{pmatrix} \qquad \nabla f(\vec{0}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$Hf(x) = \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix} = Hf(\vec{0}) = A$

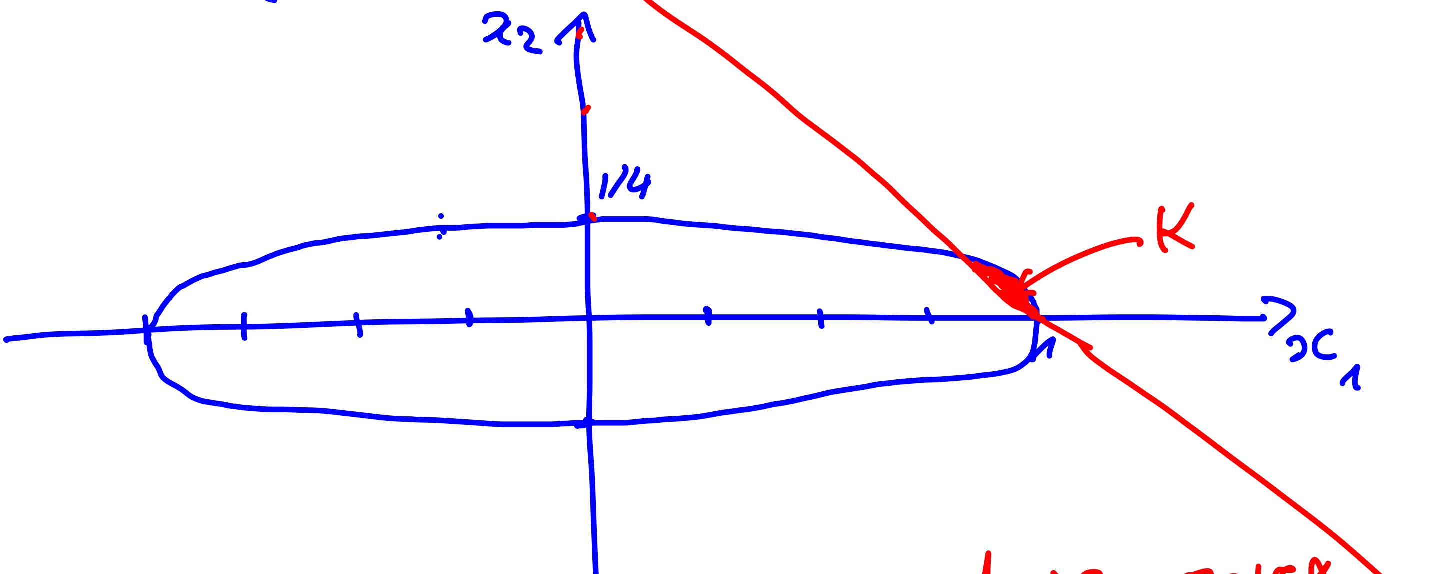
$f(x) = \frac{1}{2} \langle Ax, x \rangle$

$\det(A - \lambda I) = (4-\lambda)(6-\lambda) - 4 = \lambda^2 - 10\lambda + 20$

$\Delta = 25 - 20 = 5 \qquad \lambda = 10 \pm \sqrt{5} > 0$

$A \in S^2_{++} \implies f$ has a unique global min at $x^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$K = \{x_1^2 + 4x_2^2 \le 1 \text{ and } x_1 + x_2 \ge 1\}$$
$$= \{x_1^2 + 4x_2^2 \le 1\} \cap \{x_1 + x_2 \ge 1\}$$



intersection of 2 convex sets is convex.

isovalues of f level curves